

# Seasonality Adjustment

Natural gas risk factors (all-in and spreads) exhibit seasonal effects. Ultimately, the scenarios generated are corrected for seasonality by effectively replacing the average volatility and correlation with the seasonal equivalents. This is done by the following method:

Given average and seasonal covariance matrices,  $C_{av}$  and  $C_s$ , one can produce eigen-value decompositions as follows:

$$C_{av} = U_{av} \Lambda_{av} U_{av}^T > 0$$
$$C_s = U_s \Lambda_s U_s^T > 0$$

NOTE: A seasonal covariance matrix is one obtained by determining the covariance for returns from a given calendar month over 9 years.

Given these eigen-value decompositions, correlated deviates can be generated as:

$$\bar{x}_{av} = \Sigma_{av} A_{av} \bar{z}$$
$$\bar{x}_s = \Sigma_s A_s \bar{z}$$

Where,

$$\begin{aligned}
A_{av} &= U_{av} \Lambda_{av}^{1/2} \\
\Sigma_{av} &= \text{diag}(\sigma_{av}^i) \\
A_s &= U_s \Lambda_s^{1/2} \\
\Sigma_s &= \text{diag}(\sigma_s^i)
\end{aligned}$$

NOTE: A seasonal volatility is one obtained by determining the volatility for returns from a given calendar month over 9 years.

Therefore, one can generate seasonal scenarios from average scenarios as follows:

$$\bar{x}_s = \Sigma_s A_s A_{av}^+ \Sigma_{av}^{-1} \bar{x}_{av} \equiv P_s \bar{x}_{av}$$

$$\bar{x}_s = C_{vol} \Sigma_{av} A_s A_{av}^+ \Sigma_{av}^{-1} \bar{x}_{av} \equiv P_s \bar{x}_{av}$$

where + indicates the pseudo-inverse.

We call  $P_s$  the seasonal rotation matrix.  $C_{vol}$  is called the seasonal volatility correction factor and is given by:

$$C_{vol} = \frac{IV_s^{9yrs}}{IV_{av}^{9yrs}}$$

Where  $IV_s^{9yrs}$  and  $IV_{av}^{9yrs}$  are the average implied volatility over 9 years for a given calendar month and for the average, respectively.

In computing the rotation matrices, the following process is used:

1. make average and seasonal correlation matrices PSD
2. compute eigen-values and eigen-vectors
3. sort the eigen-values while maintaining the order of the eigen-vector matrix
4. find the eigen-value cut-off that accounts for 99% of the variance (i.e. principle components analysis)
5. convert all eigen-values that fall below the threshold to zero
6. compute:

$$P_s = \Sigma_s V_s \Lambda_s^{1/2} \Lambda_{av}^{-1/2} V_{av}^{-1} \Sigma_{av}^{-1}$$

$$P_s = \Sigma_s V_s \Lambda_s^{1/2} \Lambda_{av}^{-1/2} V_{av}^T \Sigma_{av}^{-1}$$

where,  $\Sigma_s$  is a diagonal matrix with the seasonal volatilities on the diagonal,  $\Lambda_s^{1/2}$  is a diagonal matrix with the square-root of the eigen-values of  $C_s$ ,  $V_s$  is a matrix with columns equal to the eigen-vectors of  $C_s$ ,  $\Lambda_{av}^{1/2}$  is a diagonal matrix with elements equal to the square-root of the eigen-values of  $C_{av}$ ,  $V_{av}$  is a matrix with columns equal to the eigen-vectors of  $C_{av}$ , and  $\Sigma_{av}$  is a diagonal matrix with elements equal to the average volatilities. Note that the second line of the equation is possible because of the orthogonality of  $V_{av}$ .

For all-in pipeline risk factors, both volatility and correlation are corrected for seasonality. The average volatilities and correlations are based on 1 year of historical data, while the seasonal volatility and correlation are based on 9 years of historical data. For pipeline spread risk factors, only the correlation is corrected for seasonality. The rotation matrices are output as part of the calibration package.

The seasonal volatility and correlation corrections are separated. That is, the rotation matrix is simply:

$$P_s = A_s A_{av}^+$$

The volatility is corrected with a correction factor by which the scenarios are multiplied. The correction factor is:

$$C_s = \frac{\sigma_s}{\sigma_{av}}$$

Natural gas pipeline at-the-money implied volatility risk factors receive an asymmetry (skew) adjustment. That is, the scenarios generated from a symmetric double exponential undergo a transform to create asymmetric double exponential scenarios.

The symmetric double exponential density function is given by:

$$f_s = \frac{1}{2\hat{\mu}} e^{-\frac{|x|}{\hat{\mu}}}$$

$\hat{\mu}$  is given by the average absolute deviation of the returns:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N |r_i|$$

The variance of a double exponential is given by:

$$\sigma_s^2 = 2\hat{\mu}^2$$

The asymmetric double exponential density function is given by:

$$f_a = \frac{1}{\hat{\mu}_+ + \hat{\mu}_-} \left[ e^{-\frac{x+\bar{x}_a}{\hat{\mu}_+}} 1\{x > -\bar{x}_a\} + e^{-\frac{|x+\bar{x}_a|}{\hat{\mu}_-}} 1\{x \leq -\bar{x}_a\} \right]$$

where  $\hat{\mu}_{\pm}$  is the estimate of the exponential decay parameter for positive (negative) returns and  $\bar{x}_a$  is the mode of the distribution (i.e. the point at which the positive and negative exponentials meet).

Ultimately, the transformation of the scenarios is given by:

$$X(x) = \left\{ \frac{\sqrt{2}\sigma_s}{1+R} \left[ R \left( 1 - \ln(2) + \ln\left(1 + \frac{1}{R}\right) + \frac{\sqrt{2}x}{\sigma_s} \right) - 1 \right] \right\} \text{ if } x < x_c$$

$$X(x) = \left\{ \frac{\sqrt{2}\sigma_s}{1+R} \left[ R - \left( 1 - \ln(2) + \ln(1+R) - \frac{\sqrt{2}x}{\sigma_s} \right) \right] \right\} \text{ if } x \geq x_c$$

where lower-case x is the original symmetric scenario, upper-case X is the new scenario, R is given by

$$R = \frac{\hat{\mu}_-}{\hat{\mu}_+}$$

$\sigma_s$  is the symmetric volatility and is given by:

$$\sigma_s = \frac{\hat{\mu}_+ + \hat{\mu}_-}{\sqrt{2}}$$

and  $x_c$  is given by:

$$x_c = \frac{\hat{\mu}}{1-R} \left[ (1+R) \left( \ln \left( 1 + \frac{1}{R} \right) - \ln(2) \right) - \ln \left( \frac{1}{R} \right) \right]$$

Therefore, the calibration must compute four parameters:

- The symmetric volatility  $\sigma_s$  which is already computed in section **Error! Reference source not found.**
- The symmetric decay parameter  $\hat{\mu}$
- The asymmetric positive decay parameter  $\hat{\mu}_+$
- The asymmetric negative decay parameter  $\hat{\mu}_-$

This is calculated by assuming the following linear function for the parameter R:

$$R = 0.8 + 0.025T \text{ for } T \leq 8 \text{ months}$$

$$R = 1 \text{ for } T > 8 \text{ months where } T \text{ is the contract maturity.}$$

Studies showed that the estimation of the asymmetry parameters was only reliable for the first 7 contract maturities (months) and that the asymmetry decreased in linear fashion according to the equation above (see <https://finpricing.com/lib/FiBondCoupon.html> ).

The three decay parameters are then calculated as follows for the first 7 maturities of each PL implied volatility risk factor:

$$\hat{\mu} = \frac{\sigma_s}{\sqrt{2}}$$

$$\hat{\mu}_+ = \frac{\sqrt{2}\sigma_s}{1+R}$$

$$\hat{\mu}_- = \frac{\sqrt{2}\sigma_s R}{1+R}$$